Chapter 1: Number Relationships

Factors and Multiples

A **factor** is one of the numbers you multiply in a multiplication operation. The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.

A **common factor** is a number that divides into two or more other numbers with no remainder. For example, 2 is a common factor of 8 and 12. The **greatest common factor** (GCF) is the greatest whole number that divides into two or more other whole numbers with no remainder. For example, 5 is the GCF of 10 and 25.

A **common multiple** is a number that is a multiple of two or more given numbers. For example, 6, 12, and 24 are common multiples of 2 and 3. The **least common multiple** (LCM) is the least whole number that has two or more given numbers as factors. For example, 24 is the LCM of 3 and 8.

1. List all the factors of each number.
   a) 24    b) 35    c) 64    d) 100

2. Use your answers to question 1 to determine the common factors and GCF of each pair of numbers.
   a) 24 and 64    b) 35 and 100

3. Which numbers are multiples of 4?
   20  38  300  128

4. List five common multiples of 3 and 5.

5. Determine the LCM for each set of numbers.
   a) 2 and 3    b) 3, 6, and 9

Prime and Composite Numbers

A number with only two different factors, 1 and itself, is a **prime number**. For example, \( 7 = 7 \times 1 \), so 7 is a **prime number**.

A number with more than two factors is a **composite number**. For example, \( 10 = 10 \times 1 \) and \( 10 = 5 \times 2 \), so 10 is a composite number.

The number 1 is neither prime nor composite, since \( 1 = 1 \times 1 \).
6. a) List all the prime numbers from 1 to 15.  
   b) List all the composite numbers from 1 to 15.

7. Is an odd number always a prime number? Explain.

**Powers**

A numerical expression that shows repeated multiplication is called a **power**. The power $4^3$ is a shorter way of writing $4 \times 4 \times 4$. 

A power has a **base** and an **exponent**.

8. Use a power to represent each multiplication. Calculate.
   a) $2 \times 2 \times 2$  
   b) $5 \times 5 \times 5$  
   c) $7 \times 7$  
   d) $12 \times 12 \times 12$

9. Express each number as a power.
   a) 25  
   b) 8  
   c) 81  
   d) 1000

10. a) Represent the area of this shape as a power. 
    
    3 cm 3 cm

    
    b) Represent the volume of this cube as a power. 
    
    2 cm 2 cm

**Square Roots**

The product of a whole number multiplied by itself is a **perfect square**. 

For example, 49 is a perfect square because $49 = 7 \times 7$.

A **square root** is a number that, when multiplied by itself, equals the original number. 

The square root of 49 is represented as $\sqrt{49}$. $\sqrt{49} = 7$ because $7 \times 7$ or $7^2 = 49$.

11. List all the perfect squares from 1 to 100.

12. Determine each square root.
    a) $\sqrt{64}$  
    b) $\sqrt{121}$  
    c) $\sqrt{16}$  
    d) $\sqrt{625}$

13. Calculate the dimensions of a square that has an area of 144 cm$^2$.

**Order of Operations**

**Rules for the Order of Operations (“BEDMAS”)**

<table>
<thead>
<tr>
<th>Brackets</th>
<th>For example, $(6 + 3)^2 \div (3 \times 9)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponents</td>
<td>$= 9^2 \div 27$</td>
</tr>
<tr>
<td>Divide and Multiply from left to right.</td>
<td>$= 81 \div 27$</td>
</tr>
<tr>
<td>Add and Subtract from left to right.</td>
<td>$= 3$</td>
</tr>
</tbody>
</table>

14. Evaluate each expression.
    a) $3 \times 5 - 2 + 6$  
    b) $12 - 8 \div 2^2$  
    c) $(12 + 13) \div 5 - 3$
Chapter 2: Proportional Relationships

Fractions and Decimals

A proper fraction is a part of a whole. The numerator shows how many parts of a given size the fraction represents. The denominator tells how many parts the whole set has been divided into.

A decimal is a way of writing a fraction with a denominator that is a multiple of 10.

A fraction can be written as a decimal by first expressing it as an equivalent fraction with a denominator of 10, 100, 1000, … and then representing it using the place value system.

For example, calculate $0.4 \times 3.28$.

\[
0.4 = \frac{4}{10} \quad \text{and} \quad 3.28 = \frac{328}{100}
\]

\[
0.4 \times 3.28 = \frac{4}{10} \times \frac{328}{100} = \frac{4 \times 328}{10 \times 100} = \frac{1312}{1000} = 1.312
\]

Therefore, $0.4 \times 3.28 = 1.312$.

It makes sense that 4 tenths of 3.28 is one tenth of 4 times 3.28.

4. Calculate.

a) $0.3 \times 4.7$  

b) $0.06 \times 2.19$  

c) $0.265 \times 2.48$  

d) $1.32 \times 2.006$
**Dividing with Decimals**

To divide one decimal by another without using a calculator, you can multiply the divisor by a power of 10 to get a whole number, multiply the dividend by the same power of 10, and then divide.

For example, calculate \(83.5 \div 2.5\).

\[
2.5 \times 10 = 25 \\
83.5 \times 10 = 835 \\
835 \div 25 = 33.4
\]

Therefore, \(83.5 \div 2.5 = 33.4\).

It makes sense that the number of 25 tenths (2.5) in 835 tenths (83.5) is the number of 25 ones in 835.

5. Calculate the following.
   a) \(14.56 \div 0.5\)  
   b) \(1300.512 \div 0.2\)  
   c) \(33.32 \div 3.2\)  
   d) \(517.5 \div 2.4\)

**Ratios**

A *ratio* is a way to compare two or more numbers. For example, in a group of 7 boys and 9 girls, the ratio of boys to girls is \(7:9\), or 7 to 9, or \(\frac{7}{9}\).

**Equivalent ratios** represent the same comparison. The ratios \(7:9\), \(14:18\), and \(35:45\) are equivalent ratios.

In any proportion, the number that you can multiply or divide each term in a ratio by to get the equivalent term in the other ratio is called the *scale factor*. The scale factor can be either a whole number or a decimal.

A number sentence that relates two equivalent ratios is called a *proportion*.

6. Express each comparison as a ratio.
   a) the number of blue squares to the number of white squares  
   b) the number of blue squares to the total number of squares  
   c) the number of white squares to the total number of squares

7. Which ratios are equivalent to \(15:35\)?
   \[
   5:7 \quad 18:42 \quad 9:21 \quad 3:7 \quad \frac{6}{14} \quad \frac{12}{30}
   \]

8. Determine the missing number in each proportion.
   a) \(\frac{4}{9} = \frac{\quad}{81}\)  
   b) \(\frac{24}{9} = \frac{8}{\quad}\)  
   c) \(\frac{9}{\quad} = \frac{63}{49}\)  
   d) \(\frac{\quad}{3} = \frac{14}{21}\)

**Rates**

A *rate* is a comparison of two quantities measured in different units. Unlike ratios, rates include units. For example, if Sarah ran 5 km in 2 h, then she ran at the equivalent rate of 2.5 km/h.
9. Write two equivalent rates for each comparison.
   a) 6 goals in 3 games
   b) 4 km jogged in 30 min
   c) 36 km on 3 L of gas

10. Write a proportion for each situation, and determine the missing term.
    a) In 2 h, you can earn $15.00. In 8 h, you can earn $\_
    b) Six boxes contain 90 markers. One box contains \_

**Percents**

A percent is a special ratio that compares a number to 100 using the % symbol. For example, 20 of the 100 squares are shaded. Therefore, 20% of the whole is shaded.

\[
\frac{20}{100} = 0.2 \text{ or } 20\%
\]

11. What percent of the circle is shaded?

12. Copy and complete the chart.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Ratio</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{4})</td>
<td></td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>4:20</td>
<td></td>
<td></td>
<td>20%</td>
</tr>
<tr>
<td>6:8</td>
<td>0.75</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

13. Complete each calculation.
   a) \(\frac{3}{5} = \frac{\_}{100} = \_\%\)
   b) \(\frac{21}{25} = \frac{\_}{100} = \_\%\)

14. Write each fraction as a percent.
   a) \(\frac{19}{20}\)
   b) \(\frac{1}{5}\)
   c) \(\frac{13}{25}\)
   d) \(\frac{7}{10}\)

15. Calculate.
   a) 25% of 36
   b) 15% of 160
   c) 30% of \_ = 24
   d) 10% of \_ = 14
Chapter 3: Collecting, Organizing, and Displaying Data

Reading and Drawing Graphs

Before answering any questions about the information in a graph, you need to understand the parts of the graph:

- what type of graph it is (for example, a pictograph, bar graph, histogram, line graph, scatter plot, or circle graph)
- what the title tells you about the information in the graph
- what the labels on the axes mean
- what the units for the scales are
- what the legend (if there is one) tells you

1. a) What type of graph is shown?
   b) How much was the profit from CD sales in September?
   c) How much was the profit from CD sales in November?
   d) What is the difference between the profits from DVD sales in October and December?

2. a) What type of graph is shown?
   b) What distance was travelled in the first 30 min?
   c) How long did it take to complete the 135 km trip?
   d) What happened between 30 and 40 min?
   e) Write the rate that compares the total distance travelled to the total time.

3. a) What type of graph is shown?
   b) How many books does each symbol represent?
   c) How many books were checked out of the library in January?
   d) In which months was the same number of books checked out?
   e) How many books, on average, are checked out of the library each month?
4. Last week, Joe recorded the daily high temperatures as shown.

<table>
<thead>
<tr>
<th>Day</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>15</td>
</tr>
<tr>
<td>Tuesday</td>
<td>17</td>
</tr>
<tr>
<td>Wednesday</td>
<td>21</td>
</tr>
<tr>
<td>Thursday</td>
<td>19</td>
</tr>
<tr>
<td>Friday</td>
<td>23</td>
</tr>
<tr>
<td>Saturday</td>
<td>23</td>
</tr>
<tr>
<td>Sunday</td>
<td>20</td>
</tr>
</tbody>
</table>

a) What type of graph would you use to display Joe’s data? Why?
b) Draw this graph.

Scatter Plots

A scatter plot is a graph designed to show a relationship between two variables on a coordinate grid.

5. a) What quantity is represented on the horizontal axis of this scatter plot?
b) What quantity is represented on the vertical axis of this scatter plot?
c) Does this scatter plot show a relationship between the two quantities? If so, explain what it is. If not, explain why not.

6. a) Create a scatter plot for the data in this table.
b) Describe the relationship that your scatter plot shows.

Winning Times in Women’s Olympic 100 m Sprint

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (s)</td>
<td>12.2</td>
<td>11.9</td>
<td>11.5</td>
<td>11.9</td>
<td>11.65</td>
<td>11.82</td>
<td>11.18</td>
<td>11.49</td>
<td>11.08</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (s)</td>
<td>11.07</td>
<td>11.08</td>
<td>11.06</td>
<td>10.97</td>
<td>10.54</td>
<td>10.82</td>
<td>10.94</td>
<td>10.75</td>
<td>10.93</td>
</tr>
</tbody>
</table>

Sample and Population

The total number of individuals or items is called the population.

A sample is a part of a population that is used to make predictions about the whole population.

A census is used to count and question an entire population.

Mean, Median, and Mode

The mean of a set of numbers is a description of the average number in the set. It is calculated by dividing the sum of the set of numbers by the number of numbers in the set.
The middle number in a set of ordered numbers is the **median**. When there is an even number of numbers, the median is the mean of the two middle numbers.

The number that occurs most often in a set of numbers is the **mode**. There may be more than one mode, or there might be no mode.

### 7. Determine the mean, median, and mode of each set of data.

**a)** 4, 9, 5, 9, 6, 8, 9, 11  
**b)** 2, 3, 6, 12, 6, 1, 3, 6, 4

### 8. These marks were scored by 20 students on a test.

**a)** Determine the mean, median, and mode of this set of data.
**b)** Create a sample of this set of data. Determine the mean, median, and mode of your sample.

### Stem-and-Leaf Plots

A **stem-and-leaf plot** organizes numerical data based on place values. The digits that represent the greater values are the stems. The other digits are the leaves.

For example, a basketball team scores these points in 14 games: 129, 108, 114, 125, 132, 107, 97, 127, 108, 124, 117, 94, 99, 108


Enter the scores in the stem-and-leaf plot. Use an appropriate interval for the stem. The stem-and-leaf plot of the scores is shown.

### 9. a)** Based on the data shown, how many students wrote the test?  
**b)** What was the lowest score? What was the highest score?  
**c)** How many students scored above 90%?

### 10. Determine the mean, median, and mode of the data in the stem-and-leaf plot in question 9.

### 11. The heights, in centimetres, of Arjun’s classmates are shown.

**a)** Display the heights in a stem-and-leaf plot.  
**b)** What is the median height?
Chapter 4: Patterns and Relationships

Pattern Rules for Sequences

A sequence is a list of things that are in a logical order or follow a pattern. For example, the sequence 1, 3, 5, 7, 9, … is the list of odd numbers. Each item or number in a sequence is called a term. In the sequence 1, 3, 5, 7, 9, …, the third term is 5. This sequence follows a pattern that can be described using an addition rule: “Start at 1. Add 2 to each term to get the next number in the sequence.”

1. Describe the pattern rule for each sequence. Write the next three terms.
   a) 3, 8, 13, 18, 23, …
   b) 2, 4, 8, 16, 32, 64, …
   c) 100, 96, 92, 88, 84, …

2. The pattern rule for a sequence is “Start with 7. Double the term number, and subtract 5 to get the next term in the sequence.” Write the first five numbers in the sequence.

3. The student council held a bake sale to raise money for the United Way. In the first hour, the students sold $120 worth of baked goods. Each hour after that, they sold half of the previous hour’s sales. The sale lasted for 6 h.
   a) What were their sales in the second, third, and fourth hours?
   b) What were their total sales?

Tables of Values and Scatter Plots

A table of values is an orderly arrangement of facts, usually set up in vertical and horizontal columns for easy reference.

A scatter plot is a graph designed to show a relationship between two variables on a coordinate grid. For example, the values in the following table of values are plotted on the scatter plot at the right.

<table>
<thead>
<tr>
<th>Term number</th>
<th>Picture</th>
<th>Term value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>![Picture for Term 1]</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>![Picture for Term 2]</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>![Picture for Term 3]</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>![Picture for Term 4]</td>
<td>14</td>
</tr>
</tbody>
</table>

The pattern rule is the same in both the table of values and the scatter plot: “Start with 2, and add 4 to each term.”
4. Maria and Vlad used counters to show a sequence of triangular numbers. They made the following table of values.

a) What are the fifth, sixth, and seventh triangular numbers? Explain how you know.

b) Write the pattern rule that uses an “adding on” strategy to describe how you could calculate any triangular number in this sequence.

c) Calculate the 10th triangular number.

<table>
<thead>
<tr>
<th>Term number (figure number)</th>
<th>Picture</th>
<th>Term value (number of counters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>![Picture 1]</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>![Picture 2]</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>![Picture 3]</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>![Picture 4]</td>
<td>10</td>
</tr>
</tbody>
</table>

5. a) Use a table of values to predict the number of squares you would need to build the sixth figure in this sequence.

b) Explain the pattern rule.

6. Create a scatter plot for the table of values. Use your scatter plot to determine the missing values in the table.

<table>
<thead>
<tr>
<th>Term number</th>
<th>Term value</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>17</td>
</tr>
<tr>
<td>□</td>
<td>20</td>
</tr>
<tr>
<td>□</td>
<td>23</td>
</tr>
<tr>
<td>10</td>
<td>□</td>
</tr>
<tr>
<td>15</td>
<td>47</td>
</tr>
</tbody>
</table>

7. a) These figures are the first four figures in a sequence. Make a table of values to show the number of triangles of each colour used to build each figure.

b) Draw a scatter plot to show the relationship between the red triangles in each figure and the term number.

c) Draw a scatter plot to show the relationship between the blue triangles in each figure and the term number.
Chapter 5: Measurement of Circles

Expressing Measurements in Different Units

The metre is the base unit for linear measurements in the metric system.

<table>
<thead>
<tr>
<th>1 km (kilometre)</th>
<th>1 hm (hectometre)</th>
<th>1 dam (decametre)</th>
<th>1 m (metre)</th>
<th>1 dm (decimetre)</th>
<th>1 cm (centimetre)</th>
<th>1 mm (millimetre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000 m</td>
<td>100 m</td>
<td>10 m</td>
<td>1 m</td>
<td>0.1 m or 0.01 m</td>
<td>0.01 m or 0.001 m</td>
<td></td>
</tr>
</tbody>
</table>

To rewrite a smaller unit using a larger unit, divide by the appropriate power of 10. For example, rewrite 32 mm using centimetres.

32 mm ÷ 10 = 3.2 cm

To rewrite a larger unit using a smaller unit, multiply by the appropriate power of 10. For example, rewrite 32 m using centimetres.

32 m × 10² = 3200 cm

1. Rewrite each measurement using millimetres.
   a) 52 cm = mm
   b) 68 cm = mm
   c) 71 cm = mm
   d) 73 m = mm

2. Rewrite each measurement using centimetres.
   a) 70 mm = cm
   b) 105 dm = cm
   c) 6 km = cm
   d) 317 m = cm

3. Rewrite each measurement using metres.
   a) 7 km = m
   b) 79 cm = m
   c) 62 mm = m
   d) 872 cm = m

4. Rewrite each measurement using kilometres.
   a) 345 m = km
   b) 8000 m = km
   c) 205 m = km
   d) 26 m = km

Perimeter

The distance around a 2-D shape is called its perimeter. To calculate the perimeter of a shape, add the lengths of all the sides of the shape.

5. Determine the perimeter of each shape.
   a) [Diagram]
   b) [Diagram]
6. Tony needs to build a dog pen for his puppy. The pen will be a rectangle, with sides measuring 5 m and 8 m. How much fencing does he need?

**Area**

The number of square units needed to cover the surface of a shape is its **area**.

**Area Formulas**

<table>
<thead>
<tr>
<th>Shape</th>
<th>Diagram</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>triangle</td>
<td><img src="image" alt="Triangle Diagram" /></td>
<td>$A = \frac{b \times h}{2}$</td>
</tr>
<tr>
<td>square</td>
<td><img src="image" alt="Square Diagram" /></td>
<td>$A = s^2$</td>
</tr>
<tr>
<td>rectangle</td>
<td><img src="image" alt="Rectangle Diagram" /></td>
<td>$A = l \times w$</td>
</tr>
<tr>
<td>parallelogram</td>
<td><img src="image" alt="Parallelogram Diagram" /></td>
<td>$A = b \times h$</td>
</tr>
<tr>
<td>trapezoid</td>
<td><img src="image" alt="Trapezoid Diagram" /></td>
<td>$A = (a + b) \times h \div 2$</td>
</tr>
</tbody>
</table>

7. What is the area of each shape, in square units?

   a) ![Square Grid](image)
   b) ![Trapezoid Grid](image)

8. Calculate the area of each shape.

   a) ![Rectangle Grid](image)
   b) ![Triangle Grid](image)
Chapter 6: Integer Operations

Comparing and Ordering Integers

The set of integers consists of all positive and negative whole numbers, including 0:

\[ \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \]

You can compare integers by placing them on a number line. For example, \(-3 < -1\), because \(-3\) is to the left of \(-1\) on a number line. This can also be written as \(-1 > -3\).

Opposite integers are the same distance away from 0 on a number line. For example, \(-5\) and 5 are opposite integers.

1. Identify the integer that each letter on the number line represents.

2. Draw a number line, and mark each of the following integers on it. Then list the integers from least to greatest.

\[ 9, -3, 7, 0, -5, -8, 3, -2 \]

3. Use < or > to make each statement true.

   a) \(-18 < -13\)  
   b) \(12 < -6\)  
   c) \(11 < 23\)  
   d) \(-9 < 5\)  
   e) \(-22 < -24\)  
   f) \(16 < -16\)

Adding Integers and the Zero Principle

Integer addition uses the zero principle. The zero principle shows that the sum of any two opposite integers is 0. For example, the sum of \((+1)\) and \((-1)\) is 0.

Integer addition can be modelled on a number line. Use a line pointing to the right to represent a positive integer. Use a line pointing to the left to represent a negative integer.
For example, calculate \((-6) + (+4)\).
Start at 0 on a number line. Draw a line that is 6 units long and points to the left. This represents \(-6\).
Then, starting at the point where the first line ends, draw a line that is 4 units long and points to the right. This represents \(+4\). The second line ends at \(-2\) on the number line. So, \((-6) + (+4) = -2\). Note that the overlap of the lines becomes 0, based on the zero principle.

\[ \begin{align*}
-6 & \quad \text{blue} \\
+4 & \quad \text{red}
\end{align*} \]

Integer addition can also be modelled with coloured counters, using red for positive and blue for negative. For example, calculate \((-6) + (+4)\). Use 6 blue counters to represent \(-6\) and 4 red counters to represent \(+4\).

Use the zero principle: a red counter and a blue counter together are called a zero pair because their sum is 0. Circle all the zero pairs. Since the sum of all the zero pairs is 0, you can remove these counters.

There are 2 blue counters remaining. This represents \(-2\), so \((-6) + (+4) = -2\).

4. Write the addition represented by each model, and calculate the sum.
   a) \[
   \begin{array}{c}
   \text{red}
   \end{array}
   \]
   b) \[
   \begin{array}{c}
   \text{blue}
   \end{array}
   \]
   c) \[
   \begin{align*}
   -6 & \quad \text{blue} \\
   +9 & \quad \text{red}
\end{align*} \]
   d) \[
   \begin{align*}
   -3 & \quad \text{blue}
   \end{align*} \]

5. Add using a model.
   a) \((-6) + (+10)\) e) \((-16) + (+16)\)
   b) \((-3) + (-7)\) f) \((-23) + (-37)\)
   c) \((+15) + (-12)\) g) \((+25) + (-32)\)
   d) \((-3) + (-8) + (+11)\) h) \((-25) + (-18) + (+41)\)
### Subtracting Integers

Integer subtraction can be modelled using a number line. On the number line, locate the position of the second number in the subtraction. Draw a line from this number to the position of the first number in the subtraction. The length and direction of the line gives you the answer.

For example, calculate \((-3) - (-5)\).

The line begins at \(-5\) and continues to \(-3\). The line is 2 units long, and it goes to the right. So, \((-3) - (-5) = +2\).

Integer subtraction can also be modelled using counters. Add enough zero pairs (a red counter and a blue counter) to complete the subtraction. The counters that remain after the subtraction represent the answer.

For example, calculate \((-3) - (-5)\).

\[
\begin{align*}
&\text{(\includegraphics{blue-counters} \text{ -} \includegraphics{blue-counters}}) \\
= &\text{(\includegraphics{blue-counters} \text{ -} \includegraphics{red-counters \text{ +} \includegraphics{blue-counters}})} \\
= &\text{(\includegraphics{red-counters \text{ +} \includegraphics{blue-counters}})} \\
= &\text{(\includegraphics{red-counters}}} \\
&\text{Start with 3 blue counters and 5 blue counters.} \\
&\text{Add 2 zero pairs to the 3 blue counters.} \\
&\text{Now you can subtract the blue counters.} \\
&\text{There are 2 red counters remaining. So, } (-3) - (-5) = +2.
\end{align*}
\]

6. Write the subtraction represented by each model, and calculate the difference.

a) \((\includegraphics{red-counters} \text{ -} \includegraphics{blue-counters})\)

b) \((\includegraphics{red-counters} \text{ -} \includegraphics{blue-counters})\)

c) \(\includegraphics{number-line} \text{ -} \includegraphics{number-line}\)

d) \(\includegraphics{number-line} \text{ -} \includegraphics{number-line}\)

e) \(\includegraphics{number-line} \text{ -} \includegraphics{number-line}\)

7. Subtract using a model.

a) \((+5) - (-3)\)

b) \((-6) - (-4)\)

c) \((+7) - (+12)\)

d) \((-12) - (+6)\)

e) \((-15) - (-15)\)

f) \((-26) - (+24)\)

g) \((+37) - (+42)\)

h) \((-42) - (-12)\)
Chapter 7: Transformations

Location on a Grid

A Cartesian coordinate grid is a method for describing location as the distance from a horizontal number line (the \( x \)-axis) and a vertical number line (the \( y \)-axis). The \( x \)- and \( y \)-axes intersect at \((0, 0)\), called the origin.

The location of a point is represented by an ordered pair of coordinates, \((x, y)\). The coordinates of the points on this grid are \(A(-2, 1)\), \(B(2, 2)\), \(C(-2, -2)\), and \(D(3, -1)\).

Transformations

A 2-D shape can go through various kinds of transformations, including translations, rotations, and reflections. The new shape that is created when a shape is transformed is called the image. The image is always the same size as the original shape. The vertices of the image are often labelled using the same letters as the vertices of the original shape, but with primes. \(A'\) (read “\(A\) prime”) is the image of \(A\).

Translations

A translation is the result of a 2-D shape sliding in a straight line to a new position. The shape can slide up, down, sideways, or on a slant. The shape looks the same after a translation. Only the location changes.

This is a translation of \(\triangle ABC\) 4 units to the right.

This is a translation of \(\triangle ABC\) 4 units to the right and 2 units down.

This is not a translation of \(\triangle ABC\), because the shape looks different.

1. Which shape is a translation of \(A\)?

2. Describe the translation of \(\triangle DEF\) to \(\triangle D'E'F'\) in terms of how many units up, down, to the left, or to the right \(\triangle DEF\) moved.
3. Copy parallelogram \(ABCD\) onto grid paper. Draw its image after a translation 2 units to the right and 3 units down.

4. Which shape is not a rotation of \(A\)?

5. Describe the rotation of rectangle \(ABCD\) to rectangle \(A'B'C'D'\) in terms of how many degrees and in which direction \(ABCD\) was rotated.
Reflections

A reflection is the result of a flip of a 2-D shape over a line of reflection. Each point in the shape is flipped to the opposite side of the line of reflection, but stays the same distance from the line. This is a reflection of \( \triangle ABC \) in a vertical line of reflection.

7. Which shape is a reflection of A?

8. Copy \( PQRS \) onto grid paper. Draw its image after a reflection in the line \( LR \).

Similarity and Congruence

Two shapes that are the same shape are similar. Two shapes that are the same size and shape are congruent.

9. a) Identify the congruent shapes.
   b) Identify the similar shapes.
Chapter 8: Equations and Relationships

Algebraic Expressions

An algebraic expression is the result of applying arithmetic operations to numbers and variables. For example, in the formula for the area of a rectangle, \( A = l \times w \), the variables \( A \), \( l \), and \( w \) represent the area, length, and width of the rectangle. The algebraic expression \( l \times w \) shows the calculation.

1. a) What stays the same and what changes in this sequence?
   
b) Describe the sequence in words.
   
c) Write an algebraic expression that describes the total number of counters in each term of this sequence.

2. a) Describe, in words, the sequence 1, 4, 9, 16, 25, ….
   
b) Write an algebraic expression that describes how to calculate each term in this sequence using its term number.

3. The algebraic expression \( 3b + 2 \) represents a pattern rule. Draw possible figures for the first three terms in the pattern.

4. Evaluate each expression for \( c = 6 \) and \( d = 2 \).
   
a) \( 8c \)  
b) \( 2d - 9 \)  
c) \( c^2 + d \)

5. Write an algebraic expression for each phrase.
   
a) the cost of a number of hot dogs that are $2 each
   
b) the length of a line that increases by 6 units every time
   
c) the total value of a number of quarters and dimes

Solving Equations by Inspection

An equation is a mathematical statement that two expressions are equal. For example, \( 5x + 3 = 8 \).

The solution to an equation is the value of the variable that makes the equation true. One way to solve an equation is by inspection, which means by examining it carefully.

For example, solve \( 5b - 2 = 13 \). If you add 2 to both sides, then \( 5b \) must equal 15. Since \( 5 \times 3 = 15 \), then \( b = 3 \).

When you solve an equation, check the solution by substituting it into the equation to see if it makes the equation true.

Check:

<table>
<thead>
<tr>
<th>Left side</th>
<th>Right side</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5b - 2 )</td>
<td>13</td>
</tr>
</tbody>
</table>

The equation is true when \( b = 3 \), so this solution is correct.
6. Use inspection to solve each equation. Check your solution.
   a) \( n + 5 = 9 \)  
   b) \( c - 6 = 4 \)  
   c) \( 6p = 54 \)  
   d) \( 2x + 4 = 10 \)  
   e) \( 5v - 2 = 48 \)  
   f) \( 9s - 4 = 68 \)

7. To rent a lawn mower, a company charges a fixed rate of $5, plus $2 per day.
   a) Write an equation that describes the cost \( c \) of renting a lawn mower for a certain number of days \( d \).
   b) Write an equation that represents the number of days you can rent a lawn mower for $25.
   c) Solve your equation and check your solution.

**Solving Equations by Systematic Trial**

To solve an equation by systematic trial, you need to predict the solution. Substitute your prediction into the equation, and calculate the result. If the result is too low, then increase your prediction. If the result is too high, then decrease your prediction. Repeat this process until you have the correct solution.

For example, solve \( 8k - 12 = 108 \) using systematic trial.

<table>
<thead>
<tr>
<th>Predict ( k )</th>
<th>Evaluate ( 8k - 12 )</th>
<th>Is it equal to 108?</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>8(20) - 12 = 160 - 12 = 148</td>
<td>No, it's too high.</td>
</tr>
<tr>
<td>12</td>
<td>8(12) - 12 = 96 - 12 = 84</td>
<td>No, it's too low.</td>
</tr>
<tr>
<td>15</td>
<td>8(15) - 12 = 120 - 12 = 108</td>
<td>I've solved it!</td>
</tr>
</tbody>
</table>

8. Use systematic trial to determine the value of the variable in each equation.
   a) \( c - 25 = 45 \)  
   b) \( 12d = 156 \)  
   c) \( 5 + 4c = 61 \)  
   d) \( 88 = 4g - 8 \)

9. The formula for the area of a triangle is \( A = \frac{b \times h}{2} \).
   a) Determine the value of \( b \) in centimetres, if \( A = 12 \) cm\(^2\) and \( h = 4 \) cm.
   b) Determine the value of \( h \) in centimetres, if \( A = 26 \) cm\(^2\) and \( b = 13 \) cm.
Chapter 9: Fraction Operations

Modelling Fractions

A variety of models can be used to represent fractions.
For example, each model represents the fraction \( \frac{3}{8} \).

Adding Fractions

To add fractions, the fractions need to have the same denominator. If necessary, rename the fractions so that they have a common denominator.
A common denominator is a common multiple of the two denominators.
The numerator of the answer is the sum of the numerators. The denominator of the answer is the common denominator.

For example, add \( \frac{2}{5} + \frac{1}{10} \) using fraction strips.
Represent each fraction with a fraction strip.
Align the end of one shaded region with the beginning of the other shaded region.

To get a common denominator, rename the \( \frac{2}{5} \) fraction strip as \( \frac{4}{10} \). The fraction strips show that \( \frac{2}{5} + \frac{1}{10} = \frac{5}{10} \), or \( \frac{1}{2} \).

Now add \( \frac{2}{3} + \frac{1}{6} \) using number lines.
Use a number line marked in sixths.
Rename \( \frac{2}{3} \) as \( \frac{4}{6} \). Draw arrows to show that \( \frac{4}{6} + \frac{1}{6} = \frac{5}{6} \).

1. Write the fraction addition, and calculate the sum.
   a) \[
   \begin{array}{c}
   \frac{1}{4} \\
   \frac{1}{8} \\
   \frac{1}{10}
   \end{array}
   \]
   b) \[
   \begin{array}{c}
   \frac{1}{3} \\
   \frac{1}{4}
   \end{array}
   \]

2. Add using a model.
   a) \( \frac{2}{5} + \frac{3}{5} \)   b) \( \frac{7}{8} + \frac{3}{8} \)   c) \( \frac{3}{4} + \frac{1}{2} \)   d) \( \frac{2}{5} + \frac{3}{4} \)

3. Mark has \( \frac{3}{8} \) of a tank of gas. He adds another \( \frac{1}{2} \) of a tank of gas. Is his tank full? Explain.
Subtracting Fractions

To subtract fractions, use a common denominator. The numerator of the answer is the difference between the numerators. The denominator of the answer is the common denominator.

For example, subtract $\frac{2}{5} - \frac{1}{10}$ using fraction strips.

Represent each fraction with a fraction strip. Align the ends of the fraction strips.

 Rename the $\frac{2}{5}$ fraction strip as $\frac{4}{10}$. The two fraction strips show that the difference between $\frac{2}{5}$ and $\frac{1}{10}$ is $\frac{3}{10}$.

Subtract $\frac{7}{4} - \frac{2}{3}$ using a number line.

Use a number line marked in 12ths. Rename $\frac{7}{4}$ as $\frac{21}{12}$, and rename $\frac{2}{3}$ as $\frac{8}{12}$.

There are 13 spaces between $\frac{8}{12}$ and $\frac{21}{12}$. Each space is $\frac{1}{12}$, so $\frac{21}{12} - \frac{8}{12} = \frac{13}{12}$.

4. Write the fraction subtraction, and calculate the difference.

a) $\frac{1}{3} - \frac{1}{3}

b) $\frac{1}{2} - \frac{1}{3}$

5. Subtract using a model.

a) $\frac{4}{5} - \frac{3}{5}$

b) $\frac{7}{8} - \frac{5}{8}$

c) $\frac{3}{4} - \frac{1}{2}$

d) $\frac{3}{4} - \frac{1}{5}$

6. Sarah’s pitcher of juice is $\frac{7}{8}$ full. She pours out a full glass. The glass holds $\frac{1}{4}$ of a full pitcher. How full is Sarah’s pitcher now?
Adding Mixed Numbers

A mixed number is made up of a whole number and a fraction, such as $3 \frac{1}{4}$.

To add mixed numbers, add the whole numbers and fractions separately.

For example, calculate $2 \frac{2}{3} + 1 \frac{1}{2}$ using a grid and counters.

The common denominator for $\frac{2}{3}$ and $\frac{1}{2}$ is 6, so each rectangle has 6 squares. First add the whole numbers.

$2 + 1 = 3$

To add the fractions, rename them with a common denominator.

$$\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6}$$

$$= \frac{7}{6}, \text{ or } 1 \frac{1}{6}$$

To get the final answer, add the whole number and fraction sums.

$$3 + 1 \frac{1}{6} = 4 \frac{1}{6}$$

Calculate $2 \frac{2}{3} + 1 \frac{1}{2}$ using fraction strips.

Add the whole numbers. $2 + 1 = 3$

Make a $\frac{2}{3}$ fraction strip and a $\frac{1}{2}$ fraction strip. Since 6 is the common denominator for $\frac{2}{3}$ and $\frac{1}{2}$, divide the fraction strips into sixths.

Rename the fractions and add.

$$\frac{4}{6} + \frac{3}{6} = \frac{7}{6}, \text{ or } 1 \frac{1}{6}$$

To get the final answer, add the whole number and fraction sums.

$$3 + 1 \frac{1}{6} = 4 \frac{1}{6}$$

7. Write the addition of mixed numbers, and calculate the sum.

a) b)
8. Add using a model.
   a) \( \frac{2}{3} + 5\frac{1}{3} \)  
   b) \( 5\frac{1}{2} + 3\frac{1}{4} \)  
   c) \( 2\frac{3}{8} + 2\frac{1}{4} \)

**Subtracting Mixed Numbers**

Calculate \( 5 - 1\frac{3}{10} \) using a number line.

\[ 1\frac{3}{10} + n = 5 \]

Determine how much you need to add to \( 1\frac{3}{10} \) to get to 5. Use a number line divided into 10ths.

\[
\begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10
\end{array}
\]

The distance from \( 1\frac{3}{10} \) to 2 is \( \frac{7}{10} \). The distance from 2 to 5 is 3.

Therefore, the total distance is \( \frac{7}{10} + 3 \), or \( 3\frac{7}{10} \). So, \( 5 - 1\frac{3}{10} = 3\frac{7}{10} \).

Calculate \( 5 - 1\frac{3}{10} \) using fraction strips.

Represent each number with fraction strips. Line up the fractions to compare the numbers. Then figure out the difference.

\[
\begin{array}{cccccccc}
\text{\( 1 \)} & \text{\( 1 \)} & \text{\( 1 \)} & \text{\( 1 \)} & \text{\( 1 \)} & \text{\( 1 \)}
\end{array}
\]

The difference is \( 3\frac{7}{10} \), so \( 5 - 1\frac{3}{10} = 3\frac{7}{10} \).

9. Write the subtraction, and calculate the answer.
   a)
   \[
   \begin{array}{cccccccc}
   0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10
   \end{array}
   \]

   b)
   \[
   \begin{array}{cccccccc}
   0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10
   \end{array}
   \]

10. Subtract using a model.
    a) \( 5 - \frac{1}{3} \)  
    b) \( 6 - 3\frac{3}{4} \)  
    c) \( 6 - 2\frac{1}{5} \)

11. Phil’s class ordered 6 large pizzas. His class ate \( 4\frac{7}{8} \) of the pizzas. How much is left?
Multiplying a Whole Number by a Fraction

Multiply $4 \times \frac{5}{12}$ using grids and counters.

The denominator is 12, so use 3-by-4 rectangles. Show 4 sets of $\frac{5}{12}$. Move 7 counters to fill the empty squares in one rectangle. Move the remaining counters to fill as many squares as possible in another rectangle. The counters fill $1 \frac{8}{12}$ squares. Therefore, $4 \times \frac{5}{12} = 1 \frac{8}{12}$, or $1 \frac{2}{3}$.

Multiply $5 \times \frac{1}{6}$ using fraction strips.

$5 \times \frac{1}{6}$ means 5 sets of $\frac{1}{6}$.
Put five $\frac{1}{6}$ strips together.
Together, they make $\frac{5}{6}$.
This shows that $5 \times \frac{1}{6} = \frac{5}{6}$.

Multiply $4 \times \frac{5}{2}$ using a number line.

$4 \times \frac{5}{2}$ is 4 sets of $\frac{5}{2}$. Use a number line marked in halves.
Draw 4 arrows. Each arrow shows $\frac{5}{2}$.
The arrows show that $4 \times \frac{5}{2} = \frac{20}{10}$ or 10.

12. Write the multiplication question, and calculate the answer.

13. Multiply using a model.

<table>
<thead>
<tr>
<th></th>
<th>a) $3 \times \frac{1}{2}$</th>
<th>b) $6 \times \frac{3}{4}$</th>
<th>c) $4 \times \frac{3}{8}$</th>
</tr>
</thead>
</table>

14. Every school day, Akiko has a music class that lasts $\frac{3}{4}$ of an hour.
There are 20 school days in a month. How many hours of music lessons does Akiko have each month?
Chapter 10: Angles and Triangles

Measuring and Constructing Angles

An angle is formed where two rays intersect at a point called the vertex. It is the amount of “turn,” in degrees, that is needed to turn one ray onto the other. When naming an angle, the letter that represents the vertex is placed in the middle of the letter sequence, or it can be used on its own. For example, this angle can be called ∠XYZ, ∠ZXY, or ∠Y.

A protractor can be used to measure and construct angles. To measure an angle, place the 0° line of the protractor along one ray, with its centre over the vertex. Look at the scale on the edge to see how many degrees the angle is. For example, ∠ABC is 45° and ∠DBC is 140°.

To construct an angle, start by drawing one ray. Then place the 0° line of the protractor along the ray, with its centre at the start of the ray. On the scale of the protractor, locate the degree that you want the angle to be. Mark a dot on the paper at this degree. Then draw a line from the start of the first ray through this dot.

Angles of different measurements have different names.

<table>
<thead>
<tr>
<th>Angle measurement</th>
<th>Name of angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>less than 90°</td>
<td>acute</td>
</tr>
<tr>
<td>90°</td>
<td>right</td>
</tr>
<tr>
<td>between 90° and 180°</td>
<td>obtuse</td>
</tr>
<tr>
<td>180°</td>
<td>straight</td>
</tr>
<tr>
<td>between 180° and 360°</td>
<td>reflex</td>
</tr>
</tbody>
</table>

1. Measure each angle using a protractor.

   a) [Diagram of angle A]
   b) [Diagram of angle F]

2. Name each type of angle in question 1.

3. Use a protractor to construct each angle.

   a) 30°   b) 150°   c) 85°   d) 210°
### Classifying Triangles and Quadrilaterals

Triangles can be named based on their angles.

<table>
<thead>
<tr>
<th>Name of triangle</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>acute triangle</td>
<td>All angles are less than 90°.</td>
<td><img src="image" alt="Acute Triangle" /></td>
</tr>
<tr>
<td>equilateral triangle</td>
<td>All angles are 60°.</td>
<td><img src="image" alt="Equilateral Triangle" /></td>
</tr>
<tr>
<td>obtuse triangle</td>
<td>One angle is greater than 90°.</td>
<td><img src="image" alt="Obtuse Triangle" /></td>
</tr>
<tr>
<td>right triangle</td>
<td>One angle is 90°.</td>
<td><img src="image" alt="Right Triangle" /></td>
</tr>
</tbody>
</table>

Triangles can also be named based on the lengths of their sides.

<table>
<thead>
<tr>
<th>Name of triangle</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>equilateral triangle</td>
<td>All sides are equal lengths.</td>
<td><img src="image" alt="Equilateral Triangle" /></td>
</tr>
<tr>
<td>isosceles triangle</td>
<td>Two sides are equal lengths.</td>
<td><img src="image" alt="Isosceles Triangle" /></td>
</tr>
<tr>
<td>scalene triangle</td>
<td>No sides are equal.</td>
<td><img src="image" alt="Scalene Triangle" /></td>
</tr>
</tbody>
</table>

4. Name each triangle based on their angles and lengths of sides.

a) ![Triangle A](image)  
   - 60°  
   - 42°  
   - 78°

b) ![Triangle B](image)  
   - 30°

c) ![Triangle C](image)  
   - 30°  
   - 115°  
   - 35°
Quadrilaterals are named based on the characteristics of their angles and sides. The following flow chart will help you identify quadrilaterals.

5. Name each quadrilateral.
   a)  
   b)  
   c)  

START

Does it have four right angles?

YES

Are all the sides the same length?

YES

square

YES

Are there two pairs of parallel sides?

YES

rhombus

NO

Are all the sides the same length?

NO

rectangle

NO

Is there a pair of parallel sides?

YES

parallelogram

NO

Are all the sides the same length?

NO

trapezoid

quadrilateral with no special properties

Review of Essential Skills from Grade 7: Chapter 10  457
Chapter 11: Geometry and Measurement Relationships

Surface Area

The surface area of a 3-D object is the total area of all the faces of the object, including the base.

You can use a net to determine the surface area of an object. A net is a 2-D pattern that can be folded to create a 3-D shape.

For example, the surface area of the box at the right is 22 square units.

To determine the surface area of a prism, calculate the areas of the top and bottom, and the areas of each rectangular side face. Then add the areas.

1. Use this net to determine the surface area of the folded-up prism.

2. Sketch a net of each prism. Then calculate the surface area of the prism.

   a) 
   b) 
   c) 
   d) 
   e) 
   f)
Volume

The volume of a 3-D object is the total amount of space that is occupied by the object.

For example, the volume of this box is 6 cubes.

To determine the volume of a prism, multiply the area of the base by the height of the prism.

\[ \text{Volume} = (\text{area of base}) \times (\text{height}) \]

3. Calculate the volume of each prism. Note that \( A \) represents the area of the base of the prism.

\[ \begin{align*}
\text{a) } & A = 16 \text{ cm}^2 \\
\text{b) } & A = 40 \text{ cm}^2 \\
\text{c) } & A = 30 \text{ m}^2 \\
\text{d) } & A = 64 \text{ cm}^2 \\
\text{e) } & \\
\text{f) } & 
\end{align*} \]

4. Use each net to determine the volume of the folded-up prism.

\[ \begin{align*}
\text{a) } & \\
\text{b) } & 
\end{align*} \]
Chapter 12: Probability

Probability

Probability is a number between 0 and 1 that tells the likelihood of something happening. Sometimes you can conduct an experiment, such as tossing a coin or spinning a spinner, to determine probability.

A possible outcome is a single result that can occur in a probability experiment. For example, getting Heads when tossing a coin is a possible outcome.

The favourable outcome is the desired result in a probability experiment. For example, if you spin a coloured spinner to see how often the red section comes up, then red is the favourable outcome.

An event is a set of one or more outcomes for a probability experiment. For example, if you roll a cube with the numbers 1 to 6, the event of rolling an even number has the outcomes 2, 4, or 6.

The experimental probability of an event is the measure of the likelihood of the event, based on data from an experiment. It is calculated using this ratio:

\[
\frac{\text{number of trials in which event occurred}}{\text{total number of trials in the experiment}}
\]

The theoretical probability of an event is the measure of the likelihood of the event, calculated using this ratio:

\[
\frac{\text{number of favourable outcomes for the event}}{\text{total number of possible outcomes}}
\]

The probability of an event can be expressed as a fraction, a decimal, or a percent. The probability of an event is often written as \(P(X)\), where \(X\) is a description of the event. For example, if \(P(H)\) represents the probability of tossing a coin and getting Heads, then \(P(H) = \frac{1}{2}\) or 0.5 or 50%.

1. Nesrine conducted an experiment in which she tossed two quarters together 50 times. The following chart shows her results.

<table>
<thead>
<tr>
<th>Event</th>
<th>Number of occurrences</th>
</tr>
</thead>
<tbody>
<tr>
<td>both coins Heads</td>
<td>10</td>
</tr>
<tr>
<td>one coin Heads and the other Tails</td>
<td>25</td>
</tr>
<tr>
<td>both coins Tails</td>
<td>15</td>
</tr>
</tbody>
</table>

a) Determine Nesrine’s experimental probability for each event.

b) Determine the theoretical probability of each event.

c) Which experimental probability is the same as the theoretical probability?
2. Determine the theoretical probability of each event for the following situations. Write the probability as a fraction, a decimal, and a percent.

a) Alok spins a spinner numbered from 1 to 10. The 10 sections of the spinner are equal.
   i) \( P(\text{spinning 5}) \)
   ii) \( P(\text{spinning a multiple of 3}) \)
   iii) \( P(\text{spinning a prime number}) \)
   iv) \( P(\text{spinning 11}) \)

b) Ellen rolls a 20-sided die numbered from 1 to 20.
   i) \( P(\text{rolling a 15}) \)
   ii) \( P(\text{rolling an even number}) \)
   iii) \( P(\text{rolling a number divisible by 5}) \)

c) Paul selects one coin.
   i) \( P(\text{dime}) \)
   ii) \( P(\text{copper coloured coin}) \)
   iii) \( P(\text{silver coloured coin}) \)

Tree Diagrams

A tree diagram is a way to record and count all the possible combinations of events. For example, the tree diagram at the right shows all the possible outcomes of a three-child family.

This tree diagram shows that there are eight possible outcomes. You can use it to determine probabilities.

For example,
\[ P(\text{all 3 children are boys}) = \frac{1}{8} \]

3. Use the tree diagram at the right to determine each probability.

a) \( P(3 \text{ girls}) \)

b) \( P(1 \text{ boy and 2 girls}) \)

c) \( P(\text{at least 1 girl}) \)

d) \( P(\text{all boys or all girls}) \)

4. Indira spins the spinner shown and rolls a die.

a) Create the tree diagram that shows all the possible outcomes for one spin and one roll.

b) Determine each probability below.
   i) \( P(\text{the spinner is red and the die is 5}) \)
   ii) \( P(\text{the spinner is green and the die is even}) \)
   iii) \( P(\text{the spinner is orange or yellow and the die is greater than 3}) \)
   iv) \( P(\text{the spinner is not red and the die is a multiple of 2}) \)

c) Explain why \( P(\text{the spinner is yellow and the die is 7}) = 0 \).

5. Bill rolls a pair of dice and calculates the sum of the two numbers.

a) List all the possible sums.

b) Draw a tree diagram that shows all the possible outcomes.

c) Determine the probability that Bill will toss a sum of 7.